The Demography of Health and Education

Alternative and new methods to estimate (healthy) life expectancy for subpopulations

Marc Luy
Outline of this lecture

- Indirect estimation methods
- Traditional Orphanhood Method
- Modified Orphanhood Method (MOM)
- Longitudinal Survival Method (LSM)
Indirect estimation methods
Main application: Populations with incorrect, incomplete or not existing data

Data base of indirect methods: specific questions in censuses or special surveys (WFS, DHS, national research projects)

Information from interviews with questions about
- the parents ("orphanhood method") → adult mortality
- the partner ("spouse survival technique") → adult mortality
- the siblings ("sibl. surv. tech.", "sisterhood m.") → adult and maternal mort.
- the children ("own child method") → child mortality

Goal: retrospective analysis of the past
Under-five years mortality in Ethiopia

Source: Luy 2015
Combination of indirect estimates (global indicators) for child and adult mortality to derive complete life table

Source: Newell 1988

Survival Rate

Child mortality (0-5)

Adult Mortality (25-55)

Age

fitted

observed

Source: Newell 1988
Model Life Tables

Most important model life table systems:

- UN-Tables (1955, 1956)
- Coale & Demeny (1966, 1983)
- Lederman (1969)
- OECD-Tables (1980)
- UN-Tables (1982)
- Brass (1969, 1971)
- INDEPTH Tables (2004)
- Wilmoth et al. (2011)
The Model Life Table System of Coale & Demeny

CD MLT, East Pattern, Female

CD MLT, South Pattern, Female

\( e_0 = 77.5 \)

\( e_0 = 20.0 \)

Source: Coale & Demeny 1983

Summer School “The Demography of Health and Education”—Alternative and new methods to estimate (healthy) life expectancy for subpopulations (Marc Luy)
Traditional Orphanhood method
Orphanhood method: overview

- Dominating technique for the indirect estimation of adult mortality in developing countries with lack of existing population statistics
- Basic idea: the age of respondents represents the survival time of the mother or the father (since birth of respondents)
- Consequently, the proportion of respondents of a given age with mother (or father) alive, $S(n)$, approximates a survivorship ratio from an average age at childbearing, $M$, to that age plus the age of the respondents
- Implementation: transformation of this cohort survivorship ratio into period survival of a specifically derived reference period
- The traditional variants of the OM model this relation between cohort and period mortality by using different theoretical patterns of fertility, mortality (trends) and (stable) age composition, controlling for the actual pattern of childbearing
Orphanhood method: estimation

- Three kinds of information from surveys necessary:
  1. current age of respondents: \( n \)
  2. proportion of respondents with mother/father still alive: \( S(n) \)
  3. estimate of age at childbearing: \( M \)

\[
S(n) = \frac{l(M + n)}{l(M)} \rightarrow \frac{l(25 + n)}{l(25)}
\]

- Important assumption: adult mortality is not associated with the number of surviving children, including whether or not a woman/man had any children at all.
Transition 1: cohort age $M+n$ → period age $25+n$

Transition 2: cohort age $n$ → period age $n$

Transition:
cohort age $M+n$ → period age $25+n$
Orphanhood method: Brass variant

\[ \ell(25+n)/\ell(25) = W(n) \cdot S(n-5) + (1 - W(n)) \cdot S(n) \]

### Table 86. Weighting Factors, \( W(n) \), for Conversion of Proportions of Respondents with Mother Alive into Survivorship Probabilities for Females

<table>
<thead>
<tr>
<th>Age in Years</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22 (1)</td>
<td>0.420</td>
<td>0.418</td>
<td>0.404</td>
<td>0.366</td>
<td>0.303</td>
<td>0.241</td>
<td>0.125</td>
<td>0.007</td>
<td>-0.190</td>
<td>-0.368</td>
<td>0.466</td>
</tr>
<tr>
<td>23 (2)</td>
<td>0.470</td>
<td>0.489</td>
<td>0.500</td>
<td>0.485</td>
<td>0.445</td>
<td>0.401</td>
<td>0.299</td>
<td>0.186</td>
<td>-0.017</td>
<td>-0.220</td>
<td>-0.352</td>
</tr>
<tr>
<td>24 (3)</td>
<td>0.517</td>
<td>0.556</td>
<td>0.590</td>
<td>0.598</td>
<td>0.580</td>
<td>0.554</td>
<td>0.467</td>
<td>0.361</td>
<td>0.158</td>
<td>0.059</td>
<td>0.217</td>
</tr>
<tr>
<td>25 (4)</td>
<td>0.557</td>
<td>0.618</td>
<td>0.673</td>
<td>0.704</td>
<td>0.708</td>
<td>0.701</td>
<td>0.630</td>
<td>0.361</td>
<td>0.158</td>
<td>0.059</td>
<td>0.084</td>
</tr>
<tr>
<td>26 (5)</td>
<td>0.596</td>
<td>0.678</td>
<td>0.756</td>
<td>0.809</td>
<td>0.834</td>
<td>0.844</td>
<td>0.791</td>
<td>0.535</td>
<td>0.334</td>
<td>0.270</td>
<td>0.053</td>
</tr>
<tr>
<td>27 (6)</td>
<td>0.634</td>
<td>0.738</td>
<td>0.838</td>
<td>0.913</td>
<td>0.957</td>
<td>0.986</td>
<td>0.950</td>
<td>0.708</td>
<td>0.514</td>
<td>0.456</td>
<td>0.220</td>
</tr>
<tr>
<td>28 (7)</td>
<td>0.674</td>
<td>0.800</td>
<td>0.838</td>
<td>0.913</td>
<td>1.016</td>
<td>1.016</td>
<td>1.080</td>
<td>0.884</td>
<td>0.699</td>
<td>0.456</td>
<td>0.378</td>
</tr>
<tr>
<td>29 (8)</td>
<td>0.717</td>
<td>0.863</td>
<td>0.921</td>
<td>1.016</td>
<td>1.111</td>
<td>1.111</td>
<td>1.270</td>
<td>1.063</td>
<td>0.890</td>
<td>0.645</td>
<td>0.579</td>
</tr>
<tr>
<td>30 (9)</td>
<td>0.758</td>
<td>0.924</td>
<td>1.004</td>
<td>1.085</td>
<td>1.274</td>
<td>1.274</td>
<td>1.442</td>
<td>1.250</td>
<td>1.095</td>
<td>0.856</td>
<td>0.800</td>
</tr>
</tbody>
</table>

**Estimation equation:**

\[ I_f(25+n)/I_f(25) = W(n) \cdot S(n-5) + (1 - W(n)) \cdot S(n) \]

Source: Hill et al. 1983, p. 103
Orphanhood method: Timæus variant

\[ \ell(25+n)/\ell(25) = \beta_0(n) + \beta_1(n) \cdot M + \beta_2(n) \cdot S(n-5) \]

**Table 2. Coefficients for the estimation of female survivorship from the proportion of respondents with living mothers**

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \beta_0(n) )</th>
<th>( \beta_1(n) )</th>
<th>( \beta_2(n) )</th>
<th>( R^2 )</th>
<th>( CV^a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-0.2894</td>
<td>0.00125</td>
<td>1.2559</td>
<td>0.997</td>
<td>0.0015</td>
</tr>
<tr>
<td>15</td>
<td>-0.1718</td>
<td>0.00222</td>
<td>1.1123</td>
<td>0.996</td>
<td>0.0031</td>
</tr>
<tr>
<td>20</td>
<td>-0.1513</td>
<td>0.00372</td>
<td>1.0525</td>
<td>0.995</td>
<td>0.0058</td>
</tr>
<tr>
<td>25</td>
<td>-0.1808</td>
<td>0.00586</td>
<td>1.0267</td>
<td>0.993</td>
<td>0.0088</td>
</tr>
<tr>
<td>30</td>
<td>-0.2511</td>
<td>0.00885</td>
<td>1.0219</td>
<td>0.992</td>
<td>0.0126</td>
</tr>
<tr>
<td>35</td>
<td>-0.3644</td>
<td>0.01287</td>
<td>1.0380</td>
<td>0.992</td>
<td>0.0172</td>
</tr>
<tr>
<td>40</td>
<td>-0.5181</td>
<td>0.01795</td>
<td>1.0753</td>
<td>0.992</td>
<td>0.0222</td>
</tr>
<tr>
<td>45</td>
<td>-0.6880</td>
<td>0.02342</td>
<td>1.1276</td>
<td>0.993</td>
<td>0.0271</td>
</tr>
<tr>
<td>50</td>
<td>-0.8054</td>
<td>0.02721</td>
<td>1.1678</td>
<td>0.992</td>
<td>0.0400</td>
</tr>
</tbody>
</table>

Estimation equation: \( \ell(25+n)/\ell(25) = \beta_0(n) + \beta_1(n) \cdot M + \beta_2(n) \cdot S(n-5) \)

\(^a\) Coefficient of variation = root mean squared error divided by the mean of \( nP_{25} \).

Source: Timæus 1992, p. 56
Example Orphanhood Method: Bolivia, 1975

Mortality levels West and corresponding reference periods

\[
\frac{l(25+a)}{l(25)} \rightarrow 1960.6
\]

\[
\frac{l(25+a)}{l(25)} \rightarrow 1961.3
\]

\[
\frac{l(25+a)}{l(25)} \rightarrow 1962.4
\]

\[
\frac{l(25+a)}{l(25)} \rightarrow 1963.8
\]

\[
\frac{l(25+a)}{l(25)} \rightarrow 1965.3
\]

\[
\frac{l(25+a)}{l(25)} \rightarrow 1967.0
\]

\[
\frac{l(25+a)}{l(25)} \rightarrow 1961.3
\]

\[
\frac{l(25+a)}{l(25)} \rightarrow 1960.6
\]

\[
\frac{l(25+a)}{l(25)} \rightarrow 14.4
\]
Modified Orphanhood Method
Motivation for Modified Orphanhood Method (MOM)

Limited information on mortality differences by SES in many developed countries (existing studies are based on specific sub-populations and in most cases on relative risks)

Our idea: using indirect methods (IM) for estimation of adult mortality based on survey data – Italian Multipurpose Surveys of 1998 (n = 59,050) and 2003 (n = 49,451)

This approach might provide additional knowledge and new insights into mortality differences by SES because

(1) the surveys are representative for the total populations
(2) IM enable the estimation of complete life tables by SES and thus the estimation of differences in life expectancy
(3) IM enable the estimation time trends
(4) Functionality of the method can be tested
Modified Orphanhood Method (MOM): formula

\[
\left( \frac{\hat{\ell}_{33+n}}{\hat{\ell}_{30}} \right)_t = \hat{S}(n) \cdot \frac{\left( \frac{\hat{\ell}_{33+n}}{\hat{\ell}_{30}} \right)_t}{\sum_{\alpha} \hat{\omega}_x \cdot \left( \frac{p_{x+n}}{p_x} \right)}
\]

\( \hat{\ell}_{30} \) = period life table survival probability to age 30 of respondents' parents
\( \ell_{30} \) = survival probability to age 30 of official period life table
\( t \) = calendar year (reference period = average year of death of deceased parents)
\( \hat{S}(n, n + 4) \) = proportion of respondents aged \( (n, n + 4) \) with mother/father alive
\( \hat{\omega}_x \) = proportion of parents at age \( x \) at the moment of respondents' birth
\( p_x \) = cohort life table survival probability to age \( x \)
\( \bar{n} \) = average age of respondents aged \( (n, n + 4) \)
\( \alpha, \beta \) = youngest, oldest age at childbearing
The Brass Logit Life Table Model

- Basic idea: standard life table \( l(x)_S \)
- \( \text{logit } l(x) = \alpha + \beta \text{logit } l(x)_S \)
- If \( \alpha = 0 \) and \( \beta = 1 \) \( \rightarrow l(x) = l(x)_S \)
- Other values for \( \alpha \) and \( \beta \) create new life tables that deviate systematically from the standard life table
- \( \alpha = \text{level} \) and \( \beta = \text{pattern of created model life table} \)
The Brass Logit Life Table Model

Survivors at age $x$

$\alpha = -0.02, \beta = 1.00$

$\alpha = 0.02, \beta = 1.00$

Life Table 1924/26, Males

$\alpha = 0.00, \beta = 1.00$

Age $x$

0 10 20 30 40 50 60 70 80 90 100

Survivors at age $x$
The Brass Logit Life Table Model

Survivors at age $x$

Life Table 1924/26, Males

$\alpha = 0.00$, $\beta = 1.00$

$\beta = 0.70$, $\alpha = 0.00$

$\beta = 1.50$, $\alpha = 0.00$

Age $x$
Modified Orphanhood Method (MOM): estimation

\[
\left( \frac{\hat{l}_{33+n}}{\hat{l}_{30}} \right)_t = \hat{S}(n) \cdot \frac{\left( \frac{\hat{l}_{33+n}}{\hat{l}_{30}} \right)_t}{\sum_{\alpha}^{\beta} \hat{w}_x \cdot \left( \frac{p_{x+n}}{p_x} \right)}
\]

\[
\rightarrow \left( \frac{\hat{l}_{33+n}}{\hat{l}_{30}} \right) \text{transformed into complete period life tables from age 30 by using the Brass logit life table model with the life table of the reference period as standard (Brass' } \beta = 1.0 \text{ or estimated from other data sources)}
\]
Orphanhood-based estimates for life expectancy at age 35 (MOM), Italian Multipurpose Survey 1998 (green) and 2003 (red)

MOM estimates for male life expectancy at age 30 by education for the period 1984-90 according to the Italian 1998 and 2003 multipurpose surveys.

MOM estimates for male life expectancy at age 30 by occupation for the period 1984-90 according to the Italian 1998 and 2003 multipurpose surveys

Mortality by education in Italy, 1981-82

Source: Istat (1990), La mortalità differenziale secondo alcuni fattori socio-demografici, anni 1981-82; own calculations
Mortality by education in Italy, 1991-92

Life expectancy at age 30 by education in Italy, MOM estimates

The smoking epidemic model (Lopez et al. 1994)

Slightly modified version of Ramström (1997); data: Peto et al. (2006)
Longitudinal Survival Method
Longitudinal Survival Method (LSM): Motivation

• Estimation of life expectancy for specific subpopulations—and differentials between them—is a common problem for demographers

• Population statistics often do not include the required data on deaths and the population at risk, and possibilities to link mortality data with censuses are rare

• Alternative: use of longitudinal survey data with registration of deceased participants or mortality follow-ups

• The case numbers of these data sources are in most cases too small to derive age-specific death rates what prohibits the application of classic life table techniques
Approaches to estimate life expectancy on the basis of longitudinal survey data

- Proportional hazards models (Li et al. 2014; Reuser et al. 2008; Reuser et al. 2009; Reuser et al. 2011)
- Bayesian Markov chain Monte Carlo methods (Lynch and Brown 2005)
- Multi-state Markov models (Majer et al. 2011; Matthews et al. 2009),
- Hidden Markov models (Van Den Hout et al. 2009)
- Population Attributable Fraction (Preston and Stokes 2011)
- Longitudinal Survival Method (Luy et al. 2015)
Longitudinal Survival Method (LSM): approach

- LSM was inspired by the techniques of indirect mortality estimation which are used for estimating life expectancy in many developing countries.
- Idea of these indirect methods: Transformation of the reported longitudinal survival of survey respondents’ relatives into a period life table.
- Idea of the LSM: Transformation of the observed longitudinal survival of the survey respondents into a period life table.
Longitudinal Survival Method (LSM): estimation

- Required data (for each age group of the survey population):
  1. observed longitudinal survival of survey respondents (mortality follow-up)
  2. expected longitudinal survival of survey respondents (cohort life tables)
  3. corresponding period survival of the total population (period life tables)

\[
\left( \frac{\hat{\ell}_{x+z}}{\hat{\ell}_x} \right)_t = \hat{S}(\bar{x}, \bar{x} + \bar{z}) \cdot w(\bar{x} + \bar{z}, x + z)_t \cdot \frac{S_P(\bar{x}, \bar{x} + \bar{z})_t}{S_L(\bar{x}, \bar{x} + \bar{z})}
\]

- Central assumption: the relationship between cohort and period survival prevalent in the entire population applies equivalently to each subpopulation
Finally, the estimated period survivorship probabilities from age $x$ to $x+z$ are combined to one complete life table (several approaches possible)

$$
\left( \frac{\hat{\ell}_{x+z}}{\hat{\ell}_x} \right)_t = \hat{S}(\bar{x}, \bar{x} + \bar{z}) \cdot w(\bar{x} + \bar{z}, x + z)_t \cdot \frac{S_p(\bar{x}, \bar{x} + \bar{z})_t}{S_L(\bar{x}, \bar{x} + \bar{z})}
$$
Practical application of the LSM: estimation of life expectancy by education in Germany

- German Life Expectancy Survey (LES): two interview waves in 1984/86 and 1998 (mortality follow-up)
- West-sample: 3,141 women (285 deaths), 3,450 men (613 deaths)
- Education levels: low (ISCED 0-2), medium (3-4), high (5+)
- Reference year 1992 (period life table 1991/93 for West Germany)
- For practical implementation of LSM with LES see Luy et al. (2015)
Estimated probabilities of dying with the LSM in comparison to the official German life table

- **Life table 1991/93**
- **LSM estimate**
Estimated probabilities of dying by level of education with the LSM, Germany 1992

Data: Life Expectancy Survey & lebenserwartung.info, own calculation
Estimated survival functions (from age 40) by education level for West Germany, 1992

Men

Women

- Typical education gradient among both sexes
- Larger medium-high than medium-low differences
- Larger extent of differentials among men
Differences in $e(40)$ between highest and lowest education level in Austria (1991-92), German-speaking Switzerland (1990-97) and West Germany (1992)

<table>
<thead>
<tr>
<th></th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUT</td>
<td>6.0</td>
<td>3.4</td>
</tr>
<tr>
<td>SUI</td>
<td>6.3</td>
<td>3.3</td>
</tr>
<tr>
<td>GER</td>
<td>6.3</td>
<td>2.3</td>
</tr>
</tbody>
</table>

- Austria: census 1991 with 1-year mortality follow-up (Klotz and Asamer 2014)
- Switzerland: census 1990 with 7-years mortality follow-up (Spoerri et al. 2006)
Longitudinal Survival Method (LSM): Summary

• LSM is an alternative but comparatively simple demographic approach to derive life tables from survey data with mortality follow-up

• We refer to the method as “Longitudinal Survival Method” (LSM) because it is based on longitudinal survival experiences of survey respondents which are transformed into a period life table

• The applicability of the LSM is not restricted to the LES data used in this study but can be applied to all surveys with mortality follow-up
Longitudinal Survival Method (LSM): Advantages

- Application of LSM is highly flexible, it can be adjusted to the specific characteristics of the survey data and period/cohort life tables.
- Low demand on the data (e.g. no information about the time and age of deaths, no specific statistical distributions of deaths).
- Estimation of age-specific probabilities of dying.
- LSM can be used to estimate life tables for any subpopulation that can be identified in the underlying data.
- LSM can also be used to produce estimates for other periods by varying the reference life table for the transformation.
Longitudinal Survival Method (LSM): Limitations

• Possible source of bias: assumption that the relationship between cohort and period survival prevalent in the entire population applies equivalently to each subpopulation

• Dependence on quality of survey data: if the mortality follow-up is not representative for the studied population, no valid estimates can be derived with the LSM

• Application to small subpopulations requires additional adjustments (e.g. averaging, smoothing, interpolating)
Determinants of Longevity and Ageing in Good Health
Determinanten von Langlebigkeit und Altern in guter Gesundheit

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